Preparation of cluster states and *W states* with superconducting-quantum-interference-device qubits in cavity QED

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Abstract

We propose schemes to create cluster states and *W states* by many superconducting-quantum-interference-device (SQUID) qubits in cavities under the influence of the cavity decay. Our schemes do not require auxiliary qubits, and the excited levels are only virtually coupled throughout the scheme, which could much reduce the experimental challenge. We consider the cavity decay in our model and analytically demonstrate its detrimental influence on the prepared entangled states.

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Entanglement is an essential resource for testing quantum nonlocality and also for quantum information processing. Although many proposals have been put forward, the achievement of multi-partite entangled states are still challenging experimentally due to decoherence as well as the limitation of current techniques. This work focuses on a combinatory system with superconducting devices embedded in a cavity. Among a variety of qubit candidates, superconducting devices such as Josephson junction circuits [1, 2], Josephson junctions [3, 4, 5], Cooper pair boxes [6, 7], and superconducting quantum interference devices (SQUIDs) [8, 9, 10, 11, 12], have drawn particular interests due to their potential for integrated devices. On the other hand, cavities are currently considered as excellent devices to achieve multiqubit entanglement for atoms, ions, quantum dots, and other charge qubits [13, 14, 15, 16].

Our goal is to present ways for creating multi-partite entanglement of many rf SQUID qubits in cluster states and in *W states* in cavity QED. The cluster states [17] is the key ingredient in a measurement-based quantum computing (QC), i.e., one-way QC [18], with which we can carry out expected quantum gates by some single-qubit operations and detections. The general definition for the cluster state $\left|\phi_{\{k\}}\right\rangle_C$ is from the set of eigenvalue equations $K^{(a)}\left|\phi_{\{k\}}\right\rangle_C = (-1)^{\kappa_a}\left|\phi_{\{k\}}\right\rangle_C$, with $K^{(a)} = \sigma_x^{(a)} \bigotimes_{b\in nghb(a)} \sigma_z^{(b)}$, and nghb(a) specifying the sites of all the neighbors of the site a and $\kappa_a \in \{0,1\}$. It has been shown that the cluster states can be created by photons in linear optic system and by atoms going through cavities [19, 20]. While for doing a one-way QC, the flying qubits, such as the moving atoms and the photons, are not good candidates in view of an accurate manipulation on QC. In contrast, SQUID qubits embedded in cavities are always static, which would be good for one way QC if we could prepare them into cluster states. W states, whose general form is $W_n = (1/\sqrt{n})\left|n-1,1\right\rangle$ with $\left|n-1,1\right\rangle$ being all the totally symmetric states involving n-1 zeros and 1 one, are famous for their robustness to local measurement, even under qubit loss. This makes W states useful for, e.g., quantum communication based on many nodes of a network. A lot of schemes for W state preparation by atoms going through cavities have been proposed, e.g., a very recent scheme for rapidly creating multi-atom W states under the cavity decay [21]. Compared with the moving atoms, the SQUID qubits, without movement in the cavities, are obviously more suitable for this job.

One of the favorable features of our schemes is that we only virtually couple the excited level. So our implementation subspace is only spanned by the two lowest levels of the SQUIDs, which makes our scheme simpler than previous work [10, 11], and very robust to decoherence due to spontaneous emission from the excited level. Moreover, auxiliary qubits [20] are not required in our scheme, which improves the efficiency of our implementation. Furthermore, we consider in our treatment some imperfect factors, such as the cavity decay and various coupling strength of different SQUID qubits to the cavity mode and the external microwave.

For each SQUID, two lowest flux states and an excited state are employed in our model, as shown in Fig. 1 where quantum information is encoded in $|0\rangle$ and $|1\rangle$. The spacing between two neighboring SQUIDs is assumed to be much larger than the size of each SQUID ring (on the order of 10-100 μm) so that the interaction between any two SQUIDs is negligible. The SQUIDs are radiated by the cavity field and by a classical microwave pulse which are tuned respectively to $\omega_c = (\omega_{02}-\delta)$ and $\omega_{\mu w} = (\omega_{12}-\delta)$, with ω_{02} and ω_{12} resonant frequencies between levels $|0\rangle$ and $|2\rangle$, and $|1\rangle$ and $|2\rangle$, respectively. δ is the detuning. The Hamiltonian of the SQUID can be written in an usual form [10, 22], i.e., $H_{sj} = \frac{Q_j^2}{2C_j} + \frac{(\Phi_j-\Phi_{xj})^2}{2L_j} - E_J\cos\left(2\pi\frac{\Phi_j}{\Phi_0}\right)$, where the conjugate variables Φ_j and Q_j are, respectively, the magnetic flux threading the ring and the total charge on the capacitor of the jth-SQUID, with the commutation relation $[\Phi_j, Q_j] = i\hbar$. Φ_{xj} is the static external flux applied to the ring of the jth-SQUID, and $E_J = I_{cj}\Phi_0/2\pi$ is the maximum Josephson coupling energy of the jth-SQUID with I_{cj} the critical current of the junction, and $\Phi_0 = h/2e$ the flux quantum. Under the action of a single-mode cavity field $H_c = \omega_c(a^\dagger a + \frac{1}{2})$, with $\hbar = 1$ assumed, and $a^\dagger$ and a the creation and annihilation operators of the cavity mode, we have the interaction Hamiltonian H_I, $H_I = \sum_{j=1}^{N} -\frac{1}{L_j}(\Phi_j - \Phi_{xj})(\Phi_{cj} + \Phi_{\mu wj})$, where Φ_{cj} and $\Phi_{\mu wj}$ are the magnetic flux threading the ring of the jth-SQUID generated by the magnetic component $\overrightarrow{\mathbf{B}_j}(\overrightarrow{\mathbf{r}}, t)$ of the cavity field and the microwave pulse, respectively. $\Phi_{ij} = \int_{s_j} \overrightarrow{\mathbf{B}_j}(\overrightarrow{\mathbf{r}}, t)\cdot d\overrightarrow{\mathbf{S}}_j$ with $i = c, \mu w$ corresponding to the cavity field and the microwave pulse, respectively, and $j = 1, 2, \ldots, N$ regarding different qubits.

Following the steps in [10], we get to the Hamiltonian of the system in the interaction picture $H^I = \sum_{j=1}^{N} g_j a^\dagger e^{-i\delta t}\left|0\right\rangle_{jj}\left\langle 2\right| + g_j a e^{i\delta t}\left|2\right\rangle_{jj}\left\langle 0\right| + \Omega_j e^{-i\delta t}\left|1\right\rangle_{jj}\left\langle 2\right| + \Omega_j e^{i\delta t}\left|2\right\rangle_{jj}\left\langle 1\right|$.We can adiabatically eliminate the excited level $|2\rangle$ by a similar way to in [23] and reach an effective Hamiltonian, $H_I' = \sum_{j=1}^{N}\lambda_j\left(a^\dagger\left|0\right\rangle_{jj}\left\langle 1\right| + a\left|1\right\rangle_{jj}\left\langle 0\right|\right)$, where $\lambda_j = \frac{g_j\Omega_j}{\delta_j}$ and Max $\{g_j, \Omega_j\} \ll \delta_j$. Considering the influence from the cavity decay, we obtain following Hamiltonian,

$$H = \sum_{j=1}^{N}\lambda_j\left(a^\dagger\left|0\right\rangle_{jj}\left\langle 1\right| + a\left|1\right\rangle_{jj}\left\langle 0\right|\right) - i\frac{\kappa}{2}a\dagger a, \tag{1}$$

where κ is the cavity decay rate, and we have assumed that κ is much smaller than λ_j so that no quantum jump due to the cavity decay actually occurs during the time evolution under our consideration. To obtain cluster states, we assume that the N SQUIDs are prepared initially in a product state $|\psi_0\rangle = \otimes_{j=1}^{N-1} |+\rangle_j |0\rangle_N$ where $|+\rangle_j$ is the eigenstate of σ_x^j with eigenvalue 1. The cavity field is prepared in $\frac{1}{\sqrt{2}}(|0\rangle_c + i|1\rangle_c)$, which is from a resonant interaction of an ancilla qubit initially in the state $(|0\rangle - |1\rangle)/\sqrt{2}$ with a vacuum cavity. Before we get started, we assume the SQUIDs to be decoupled from the cavity field, and the microwave to remain off. By adjusting the level structure of the SQUID 1 and turning on the classical microwave to meet the condition in Fig. 1, we reach Eq. (1) with N=1. A straightforward solution of Eq. (1) could yield $|0\rangle_c|0\rangle_1 \rightarrow |0\rangle_c|0\rangle_1$, $|0\rangle_c|1\rangle_1 \rightarrow e^{-\frac{\kappa t}{4}}\{[\frac{\kappa}{4G_1}\sin(G_1 t) + \cos(G_1 t)]|0\rangle_c|1\rangle_1 - i\frac{\lambda_1}{G_1}\sin(G_1 t)|1\rangle_c|0\rangle_1\}$, $|1\rangle_c|0\rangle_1 \rightarrow e^{-\frac{\kappa t}{4}}\{[\frac{\kappa}{4G_1}\sin(G_1 t) + \cos(G_1 t)]|1\rangle_c|0\rangle_1 - i\frac{\lambda_1}{G_1}\sin(G_1 t)|0\rangle_c|1\rangle_1\}$, $|1\rangle_c|1\rangle_1 \rightarrow e^{-\frac{\kappa t}{2}}|1\rangle_c|1\rangle_1$, where $G_1 = \sqrt{\lambda_1^2 - \frac{\kappa^2}{16}}$. If we choose the evolution time of the system to be $t_1 = [\arctan(-\frac{4G_1}{\kappa}) + \pi]/G_1$, the state evolution of the system is given by $|\psi\rangle_1 = \frac{1}{2}[|0\rangle_c\left(|0\rangle_1 + e^{-\frac{\kappa t_1}{4}}\frac{\lambda_1}{G_1}\sin(G_1 t_1)|1\rangle_1\right) + i|1\rangle_c\sigma_z^1\left(e^{-\frac{\kappa t_1}{4}}\frac{\lambda_1}{G_1}\sin(G_1 t_1)|0\rangle_1 + e^{-\frac{\kappa t_1}{2}}|1\rangle_1\right)] \otimes \prod_{j=2}^{N-1}|+\rangle_j|0\rangle_N$, where $\sigma_z^1 = |1\rangle_{11}\langle 1| - |0\rangle_{11}\langle 0|$, and the state is not normalized if κ is not zero. The cluster states prepared below will be written also following this convention.

Our next step is to adjust the level structure of the SQUID 1 back to the previous situation with decoupling to the radiation field, but adjusting the level structure of the SQUID 2 to do the same job as done on the SQUID 1. So we have $|\psi\rangle_2 = \frac{1}{2\sqrt{2}}\{|0\rangle_c[|0\rangle_2|x_1\rangle + \sigma_z^1 e^{-\frac{\kappa t_2}{4}}\frac{\lambda_2}{G_2}\sin(G_2 t_2)|1\rangle_2|y_1\rangle] + i|1\rangle_c\sigma_z^2[e^{-\frac{\kappa t_2}{4}}\frac{\lambda_2}{G_2}\sin(G_2 t_2)|0\rangle_2|x_1\rangle + \sigma_z^1 e^{-\frac{\kappa t_2}{2}}|1\rangle_2|y_1\rangle]\} \otimes \prod_{j=3}^{N-1}|+\rangle_j|0\rangle_N$, where $|x_1\rangle = |0\rangle_1 + e^{-\frac{\kappa t_1}{4}}\frac{\lambda_1}{G_1}\sin(G_1 t_1)|1\rangle_1$, $|y_1\rangle = e^{-\frac{\kappa t_1}{4}}\frac{\lambda_1}{G_1}\sin(G_1 t_1)|0\rangle_1 + e^{-\frac{\kappa t_1}{2}}|1\rangle_1$ and $G_2 = \sqrt{\lambda_2^2 - \frac{\kappa^2}{16}}$ is the effective coupling regarding the SQUID 2, including the cavity decay. Step by step we may carry out above operation on the rest SQUIDs individually. After the (N-1)-th SQUID is performed by the same operation as on the first one, we have

$$|\psi\rangle_{N-1} = \frac{1}{2^{N/2}}(|0\rangle_c|x_{N-1}\rangle + i|1\rangle_c\sigma_z^{N-1}|y_{N-1}\rangle) \otimes |0\rangle_N, \tag{2}$$

where $|x_{N-1}\rangle$ and $|y_{N-1}\rangle$ are two different entangled states of the first (N-1) SQUIDs, and they follow recursive relations below $|x_{N-1}\rangle = |0\rangle_{N-1}|x_{N-2}\rangle + \sigma_z^{N-2}e^{-\frac{\kappa t_{N-1}}{4}}\frac{\lambda_{N-1}}{G_{N-1}}\sin(G_{N-1}t_{N-1})|1\rangle_{N-1}|y_{N-2}\rangle$, $|y_{N-1}\rangle = e^{-\frac{\kappa t_{N-1}}{4}}\frac{\lambda_{N-1}}{G_{N-1}}\sin(G_{N-1}t_{N-1})|0\rangle_{N-1}|x_{N-2}\rangle + \sigma_z^{N-2}e^{-\frac{\kappa t_{N-1}}{2}}|1\rangle_{N-1}|y_{N-2}\rangle$. For the Nth SQUID we stop the state evolution at $t_N = \frac{1}{G_N}[\arctan(-\frac{4G_N}{\kappa}) + \pi]$, with $G_N = \sqrt{\lambda_N^2 - \frac{\kappa^2}{16}}$, then can obtain

$$|\psi\rangle_N = \frac{1}{2^{N/2}}\left(|0\rangle_N|x_{N-1}\rangle + \frac{\lambda_N}{G_N}e^{-\frac{\kappa t_N}{4}}\sin(G_N t_N)|1\rangle_N\sigma_z^{N-1}|y_{N-1}\rangle\right) \otimes |0\rangle_c. \tag{3}$$

To check above state, we first assume $\kappa = 0$, i.e., the ideal condition. In this case, Eq. (3) reduces to the standard cluster state [17].

$$|\Phi\rangle_N = \frac{1}{2^{N/2}} \otimes_{j=1}^N \left(|0\rangle_j + |1\rangle_j\sigma_z^{j-1}\right). \tag{4}$$

To evaluate our prepared cluster state under the cavity decay, we have calculated the fidelity F and the success probability P, as shown in Fig. 2 where we considered the qubit numbers 2, 3, and 4, respectively. It is evident that both F and P are going down with the qubit number and κ. It is understandable that the preparation of a cluster state with bigger size need longer time and is thereby affected more by the cavity decay. Thus in the presence of the cavity decay, the size of the cluster states are limited. For example, if we want $F > 0.95$, in the case of $\lambda_1 = \lambda_2 = ... = \lambda$,the maximal qubit number of the cluster state is 32, corresponding to $F = 0.951$. For a cluster state with more qubit, we could created it step by step by connecting the few-qubit cluster states by quantum phase gates on the end qubits of the neighboring SQUID chains [24]. Moreover, to keep the system as stable as possible, we should not change the level spacing of each SQUID drastically. In this sense, we estimate the appearance of imperfection due to the time delay of the level spacing adjustment, i.e., the undesired phases regarding the component states of the superposition. This estimate is plotted also in Fig. 2.

From now on, we turn to the generation of W states using SQUIDs. In contrast to the cluster state preparation by individually manipulating the SQUIDS, we have to simultaneously couple all the SQUIDs in this case to the cavity

mode and to the microwave. Provided that the SQUIDs and the cavity mode are initially in the state $\prod_{j=2}^{N} |1\rangle_1 |0\rangle_j$ and the vacuum state $|0\rangle_c$, respectively, we have the time evolution of the system by straightforwardly solving Eq. (1),

$$\begin{aligned} |\psi(t)\rangle = & \prod_{j=2}^{N} |1\rangle_1 |0\rangle_j |0\rangle_c \{1 + \frac{\lambda_1^2}{A^2}[-1 + e^{-\frac{\kappa t}{4}}(\cos\frac{Bt}{4} + \frac{\kappa}{B}\sin\frac{Bt}{4})]\} \\ & + \sum_{k=2}^{N} |1_k\rangle \prod_{j=1, j\neq k}^{N} |0\rangle_j |0\rangle_c \frac{\lambda_1\lambda_k}{A^2}[-1 + e^{-\frac{\kappa t}{4}}(\cos\frac{Bt}{4} + \frac{\kappa}{B}\sin\frac{Bt}{4})] \\ & - \prod_{j=1}^{N} |0\rangle_j |1\rangle_c \frac{i4\lambda_1}{B}\sin\frac{Bt}{4} e^{-\frac{\kappa t}{4}}. \end{aligned} \tag{5}$$

where $A^2 = \sum_{j=1}^{N} \lambda_j^2$, and $B = \sqrt{16A^2 - \kappa^2}$. In order to obtain W states of (N-1) SQUIDs, we have the conditions $t = \frac{4\pi}{B}$, and $\lambda_1^2 = A'^2 e^{\frac{\kappa t}{4}}$ to be satisfied. Then the system evolves to

$$W_{N-1} = e^{-\frac{\kappa t}{8}} \sum_{k=2}^{N} \frac{\lambda_k}{A'} |1_k\rangle \prod_{j=2, j\neq k}^{N} |0\rangle_j , \tag{6}$$

where $A'^2 = \sum_{k=2}^{N} \lambda_k^2$. This is a W state of (N-1) SQUIDs with arbitrary coefficients. By choosing the same coupling for the rest (N-1) SQUIDs except the first one, we may produce a standard W state

$$W_{N-1} = e^{-\frac{\kappa t}{8}} \frac{1}{\sqrt{N-1}} \sum_{k=2}^{N} |1_k\rangle \prod_{j=2, j\neq k}^{N} |0\rangle_j , \tag{7}$$

with the success probability $P = e^{-\frac{\kappa t}{4}}$. We plot in Fig. 3 the success probability P versus cavity decay κ. From above condition for the evolving time, we have $\lambda_1 = A' e^{\kappa\pi/2B} \geq A'$, which means that for many-qubit case, the coupling strength regarding the first qubit is much larger than those of others. This also implies that, the coupling strength regarding other SQUIDs would be quite small if λ_1 is not big. Experimentally, the difference of λ_1 from other coupling strength could be made by reducing the detuning regarding the first SQUID. Alternatively, we may put the first SQUID on the antinode of the cavity standing wave (i.e., the maximum coupling), but others on the positions deviated from corresponding antinodes.

We address below the experimental feasibility of our schemes. The implementation time of our scheme should be much shorter than the cavity decay time $\kappa^{-1} = Q/2\pi\nu_c$, where Q is the quality factor of the cavity (Q= $10^6 - 10^8$ has been achieved experimentally [25], and for superconducting qubits [26]), and ν_c is the cavity field frequency. The coupling constants of the SQUID to the cavity field and to the classical microwave available at present are $g \sim 1.8 \times 10^8 s^{-1}$, and $\Omega \sim 8.5 \times 10^7 s^{-1}$, respectively [27]. So we may have an effective coupling $\lambda \sim 10$ MHz if assuming $\delta \sim 1.5$ GHz. In this case, the interaction time τ is on the order of 10^{-7} sec, much shorter than $\kappa^{-1} \sim 4\times10^{-5}$ (with $Q = 10^7$ and $\nu_c = 40GHz$). This implies that our proposed states can be of the fidelity higher than 0.99 due to $\kappa/\lambda = 4 \times 10^{-2}$.

Although optical photons in cluster states have been experimentally demonstrated some important features for one-way QC, we prefer fixed qubits, like the SQUIDs embedded in the cavity, for storing and processing quantum information. This statement is also applicable to the W state case for quantum information processing. Moreover, due to no necessity of auxiliary qubits, our scheme could much reduce the experimental challenge for cluster state preparation. For example, generation of a two-qubit cluster state requires at least five steps in [19] due to the auxiliary states involved, while only two steps are needed in our scheme. Furthermore, we only virtually couple the excited levels throughout our scheme, which, compared to [10, 11], could both simplify the implementation steps and reduce the possibility of decoherence due to spontaneous emission. We have given the analytical expressions of the prepared cluster state and W state under the cavity decay, and numerically assessed the related fidelities.

Before ending our discussion, we give few comments on our wavefunction treatment for the cavity decay. The quantum-jump approach [28] has been widely employed in solving dissipative dynamics for quantum systems, which lies in the time evolution governed by a non-Hermitian operator and interrupted by instantaneous jumps by the detection of a photon. Normally, a solution by quantum-jump approach has to be resorted to numerical calculations.

In our case, however, to prepare high-fidelity cluster states and W states, we prefer our implementation time much shorter than the cavity decay time. Therefore, we may only focus on the time evolution of the system governed by the non-Hermitian Hamiltonian Eq. (1) before the leakage of photons occurs. In this sense, our wavefunction method would be advantageous over the quantum-jump approach in the availability of analytical solution clearly demonstrating the detrimental influence from the cavity decay on the prepared states. So our solutions might be helpful in experiments for estimating the infidelity and correcting the error, particularly for implementation with cavities of comparatively low quality. This also implies that our numerical results in the last two figures are valid only within the regime of $\kappa/\lambda \ll 1$ (*e.g.,* $\kappa/\lambda \leqslant 0.1$).

In summary, we have proposed potential schemes for creating cluster states and *W states* of many SQUIDs. Fast adjustments of the level spacings of individual SQUIDs are needed in the generation of the cluster states. As no auxiliary qubits or flying qubits involved, our scheme gives good candidates for one-way QC. In the generation of the *W states, all* the SQUIDs are coupled to the radiation fields simultaneously, which results in that the cavity decay only affects the prepared states globally, instead of on the internal structure of the W state. Throughout our schemes, the excited level is only virtually coupled, and we have specifically studied the detrimental influence from the cavity decay. Our analytical results have shown clearly that the cluster states and the W states may be generated with high-fidelity only in the case of tiny cavity decay rate.

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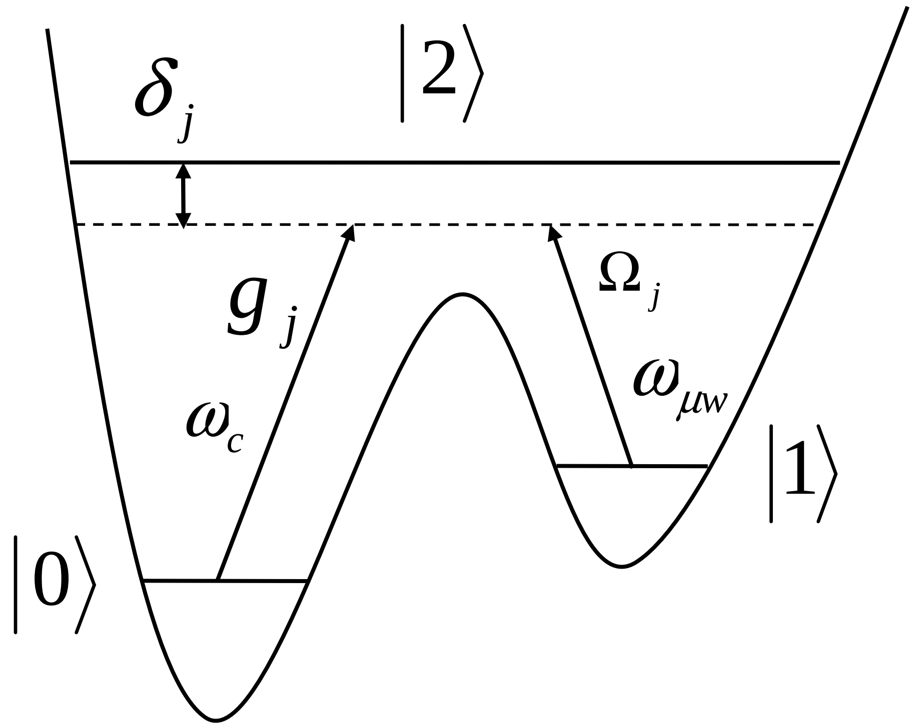
Captions of the figures

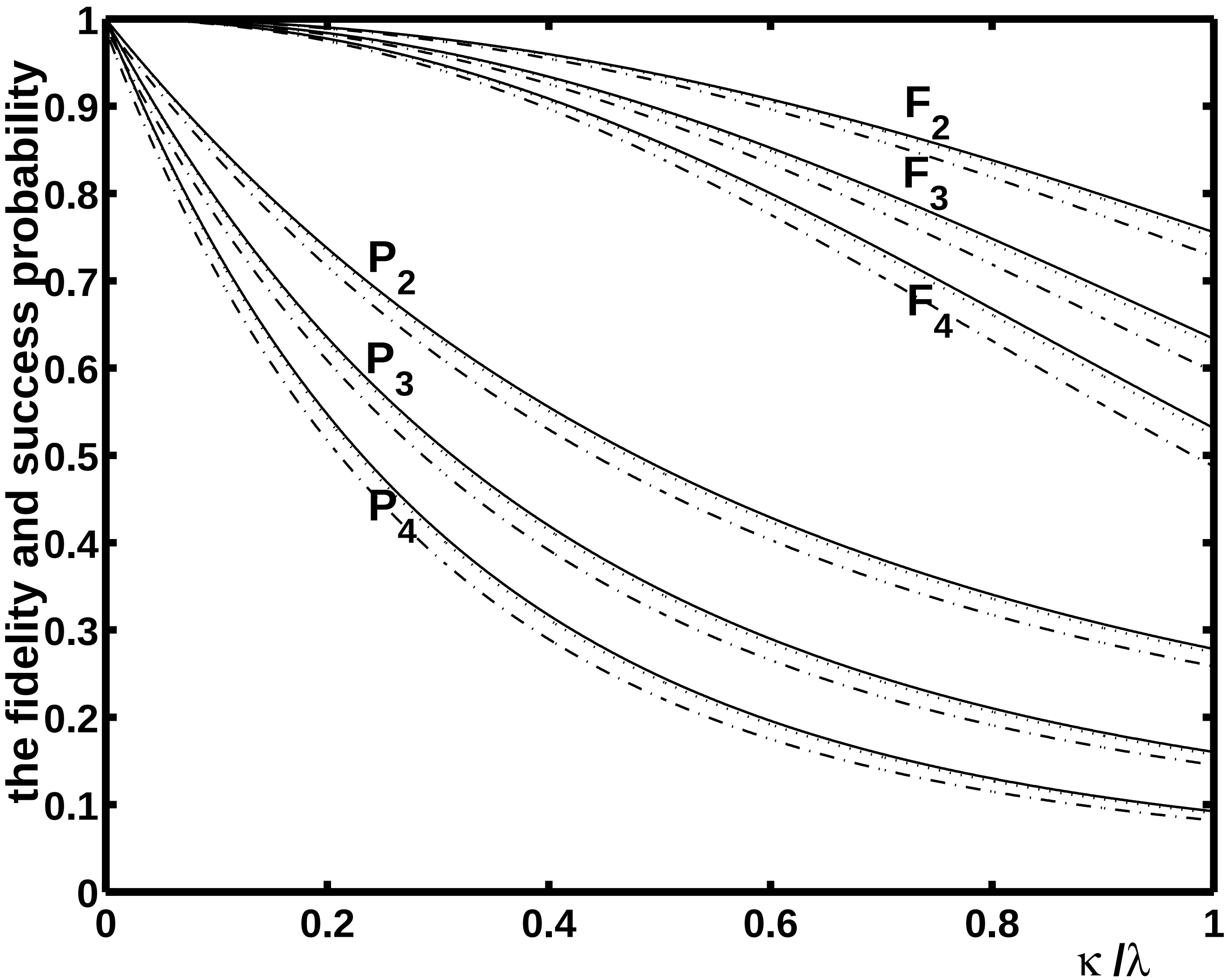
Fig. 1. The level diagram of a SQUID with three lowest levels $|0\rangle$, $|1\rangle$, and $|2\rangle$, where ω_c and $\omega_{\mu w}$ are frequecies of the cavity and the microwave, respectively. $g_j = -\frac{1}{L_j}\sqrt{(\hbar\omega_c/2\mu_0)}_j \langle 0|\,\Phi\,|2\rangle_j \int_s \overrightarrow{\mathbf{B}_j}(\overrightarrow{\mathbf{r}}) \cdot d\overrightarrow{\mathbf{S}_j}$ and $\Omega_j = -\frac{1}{L_j}{}_j \langle 1|\,\Phi\,|2\rangle_j \int_{s_j} \overrightarrow{\mathbf{B}_j}(\overrightarrow{\mathbf{r}}) \cdot d\overrightarrow{\mathbf{S}_j}$ are, respectively, the coupling constants of the jth-SQUID to the cavity field and the microwave. δ is the detuning to the excited level $|2\rangle$ by the radiation fields.

Fig. 2. The fidelity and the success probability versus the cavity decay in the generation of a cluster state, where the solid, dashed and dash-dot lines represent the consideration with the time delay $\tau_a = 0,\ 0.01\tau,\ 0.05\tau$ for adjusting the level spacing, respectively, with τ the desired time in the perfect case. The curves from the top to bottom correspond to N= 2, 3 and 4. For simplicity, we have assumed the same coupling strength for each SQUID qubits to the radiation fields, i.e.,

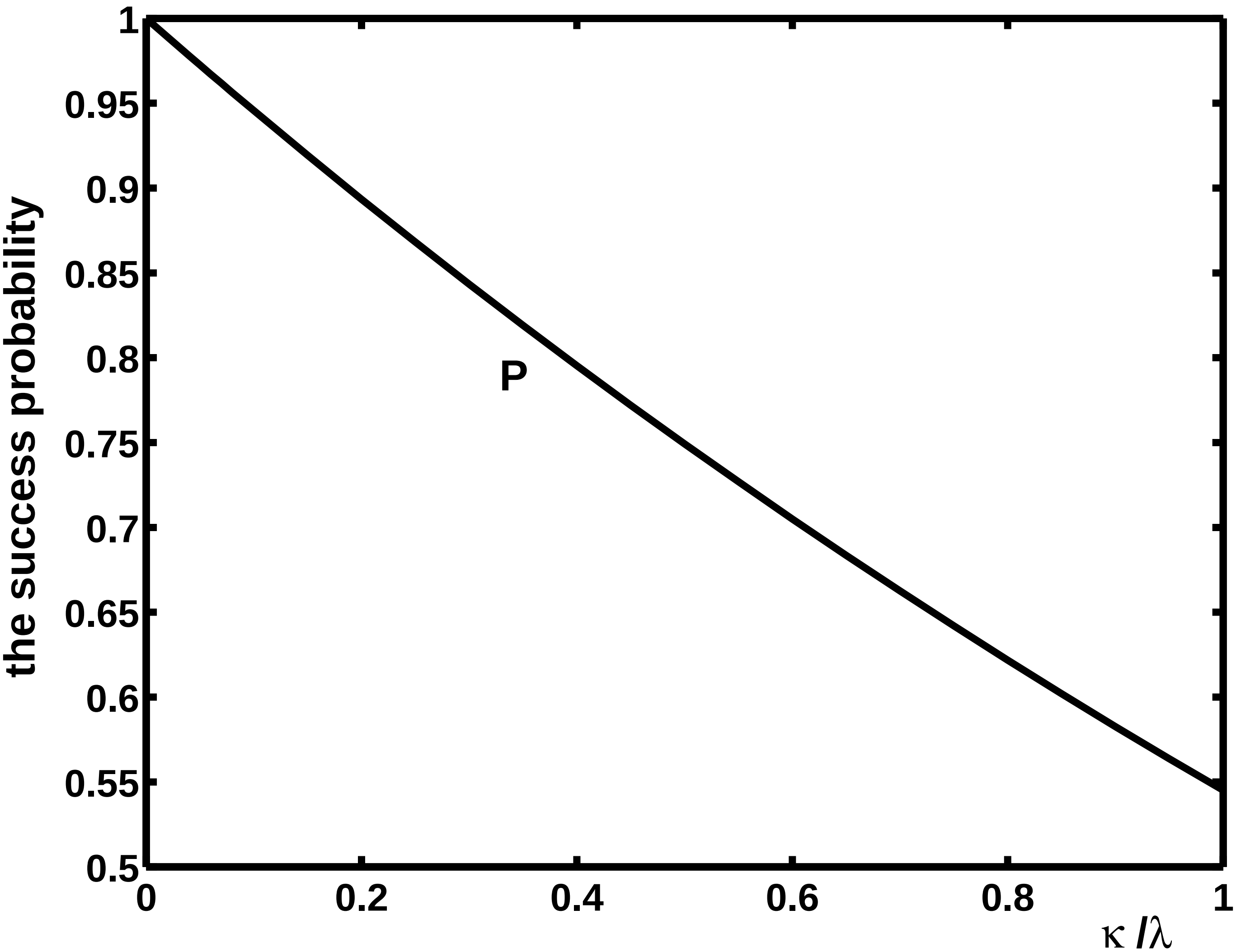
$\lambda_i = \lambda$ (i=1,2, 3, ...).

Fig. 3. The success probability versus the cavity decay in the generation of a W state, where we assume $g_1 \sim 1.8 \times 10^8 s^{-1}$, $\Omega_1 \sim 8.5 \times 10^7 s^{-1}$ and $\delta \sim 1.5 \times 10^9 s^{-1}$.


$|2\rangle$
δ_j
g_j
ω_c
Ω_j
$\omega_{\mu w}$
$|1\rangle$
$|0\rangle$



the fidelity and success probability
κ /λ
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
F2
F3
F4
P2
P3
P4


the success probability
P
κ /λ
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0
0.2
0.4
0.6
0.8
1